Measurement of the $\pi^+p$ analyzing power at 68.3 MeV

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The analyzing power $A_\lambda$ for $\pi^+p$ scattering at 68.3 MeV has been measured at the Paul Scherrer Institut with the magnetic spectrometer LEPS. The measurements cover the angular range $40^\circ < \theta_p < 70^\circ$. The protons have been polarized in a butanol target, operated in frozen spin mode. The S31 phase shift comes out by about $1^\circ$ smaller than the Koch-Pietarinen [Nucl. Phys. A 336, 331 (1980)] phase shift analysis, supporting the necessity of an alternative dispersion analysis of $\pi N$ scattering to determine the $\sigma$ term and the $\pi N$ coupling constant. [S0556-2813/96/541049-4]

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I. INTRODUCTION

Two parameters to be obtained from investigations of pion nucleon interactions are of fundamental interest: the pion nucleon coupling constants $f_{\pi NN}$ and the pion nucleon $\sigma$ term. Koch and Pietarinen [1] and Höhler [2] obtained the long-time standing ‘‘canonical’’ values of $f_{\pi NN}^2 = 0.079 \pm 0.001$ and of $\Sigma = 64 \pm 8$ MeV for the isospin even $\pi N$ on shell amplitude at the unphysical Cheng-Dashen point $\nu = 0, \tau = 2 \mu^2$. From the latter the $\sigma$ term can be determined as will be outlined below.

In recent years the Koch-Pietarinen-Höhler (KPH) value of the $\pi N$ coupling constant has been doubted and again discussed by various authors resulting in values between 0.074 and 0.081 [3–6]. There is common understanding now that the error bars of the values given in the literature are generally too low, in particular if seen in the light of the propagation of relatively high systematic errors of the experimental data used. A value of $0.076 \pm 0.003$ is well within the error band of practically all analyses.

The $\sigma$ term of pion nucleon scattering is related to the up and down quark content of the nucleon and measures the explicit chiral symmetry breaking of quantum chromodynamics:

$$\sigma_{\pi N}(t=0) = \frac{1}{2}\langle p | \hat{m}(\bar{u}u + \bar{d}d) | p \rangle$$

with

$$\hat{m} = \frac{1}{2}(m_u + m_d).$$

(1)

It may also provide hints as to the size of the nucleon matrix element $\langle p | s\bar{s} | p \rangle$ of the scalar operator $s\bar{s}$, that is, on the content of the strange sea quark pairs in the nucleon. In this way it touches fundamental questions of nucleon structure. Independent information on the strange sea quark content of the nucleon comes from measurements of deep inelastic scattering of polarized electrons and muons on polarized protons, deuterons, and $^3$He, which indicate a large nucleon matrix element of the axial vector operator $\langle p | \bar{s}\gamma_\mu \gamma_5 s | p \rangle$ resulting in a contribution of the strange sea quark pairs to the nucleon spin $\langle p | \bar{s}\gamma_\mu \gamma_5 s | p \rangle = 2 s_\mu \Delta s$ with $\Delta s = -0.10 \pm 0.03$ [7].

A low-energy theorem of current algebra relates the isospin even $\pi N$ on-shell amplitude $\Sigma = f_{\pi NN}^2 D^+(0, 2 \mu^2)$ at the unphysical Cheng-Dashen point $\nu = 0, \tau = 2 \mu^2$ to the $\sigma$ term (the overbar on $D^+$ means that the pseudovector nucleon Born term $g^2/m^2$ has been subtracted). The theorem states that in the chiral limit of vanishing quark or pion masses $(m_u = m_d = m_q = 0)$, $\Sigma = \sigma$. To test this low-energy theorem input from two sides is needed: the baryon masses can be used to determine $\sigma$ and $\pi N$ scattering data can be used to evaluate $\Sigma$. The relation between the amplitude $\Sigma$ and the $\sigma$ term for finite pion masses is given by $\Sigma = f_{\pi NN}^2 D^+(0, 2 \mu^2) = \sigma(0) + \Delta_\sigma + \Delta_K$. Gasser et al. [8] calculated $\sigma(0)$ and $\Delta_K = O(\mu^4)$ within the framework of chiral perturbation theory at the one-loop level using baryon masses as the experimental input and obtained $\sigma(0, 0) = 35$ MeV and $\Delta_K = 0.35$ MeV. For $\Delta_\sigma$ they obtain about 15 MeV, worked out by means of dispersion relations [9].

The amplitude $\Sigma(0, 2 \mu^2)$ at the unphysical Cheng-Dashen point has been evaluated by Koch, Pietarinen, and Höhler [1,2,10]. Koch and Pietarinen [1] carried out a phase-shift analysis, which extends to low energies and respects analyticity and unitarity, and consequently allows a unique extrapolation both to the $\pi N$ threshold (to determine the scattering lengths) and to the Cheng-Dashen point (to determine the amplitude $\Sigma$). The authors carefully treated the electromagnetic effects according to the method of Tromborg et al. [11,12].

At energies below 100 MeV the Koch-Pietarinen (KP) analysis is based on only a few $\pi^+p$ data points [13] and practically not on $\pi^-p$ data. The authors had to rely on the more abundant data at higher energies up to about 300 MeV [14]. Koch [10] obtained $\Sigma(0, 2 \mu^2) = 64 \pm 8$ MeV with the KP phase shifts employing dispersion relations along hyperbolic paths in the $\nu-\tau$ plane as mentioned above. In a more recent dispersion analysis in the spirit of Koch and Pietarinen, Gasser et al. [9] found a compatible value of
\[ \Sigma(0.2 \mu^2) = 60 \pm 2 \text{ MeV}. \] Using the value of \( \Delta \sigma \approx 15 \text{ MeV} \) the results obtained from the analysis of \( \pi N \) data are now \[ \Sigma(2 \mu^2) = \sigma(2 \mu^2) = 60 \text{ MeV} \] and consequently \( \sigma(0) \approx 45 \text{ MeV} \). The obvious inequality of those obtained by means of differential cross sections.

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The analyzing power determines a combination of the matrix element \( m(p|s\bar{s}|p) \) to the proton mass of the order of \( (m_s/2m) \times 10 \text{ MeV} \approx 130 \text{ MeV} \) [9].

II. EXPERIMENT AND ANALYSIS

Low-energy \( \pi N \) data have an important impact on the numbers of the \( \pi N \) coupling constant and the \( \pi N \) \( \sigma \) term [15,16]. Therefore, wide efforts have been made in recent years to improve the quality and expand the quantity of experimental data below 300 MeV. Nevertheless, some contradictions between data sets of various authors could not be resolved [6,17].

Measurements of the analyzing power of \( \pi N \) scattering seem to be very promising to clarify the intricate experimental situation. The analyzing power \( A_y \) for the scattering of pions from protons totally polarized perpendicular to the scattering plane is given by

\[
A_y = \left| \frac{d\sigma}{d\Omega} \right| \left( \frac{d\sigma}{d\Omega} + \frac{d\sigma}{d\Omega} \right) = 2 \text{ Im}(GH^*)/(|G|^2 + |H|^2).
\]

The arrows indicate the direction of the target polarization with respect to \( \vec{n} \) being the normal vector \( \vec{n} = (\vec{k} \times \vec{k'})/|\vec{k} \times \vec{k}'| \) (with \( \vec{k} \) and \( \vec{k}' \) the momenta of the incident and outgoing pions).

The analyzing power determines a combination of the spin-flip and spin-no-flip amplitudes \( G \) and \( H \) that is different from that appearing in the differential cross section and is sensitive to the smaller phases of which the \( S \) waves are of most prominent interest. Another advantage is that only ratios of numbers (cross sections) have to be measured and absolute normalization factors cancel, which allows the determination of phases with systematic errors different from those obtained by means of differential cross sections.

Angular distributions of the analyzing power for \( \pi^+ p \) elastic scattering at low energies have been measured with a polarized target by Sevior et al. [18] for energies spanning the region of the \( \Delta \) resonance (98–263 MeV). The data are in good agreement with the KP phase shifts, except for a few \( \pi^- p \) data points at 98 MeV. No measurements of the analyzing power exist below that energy. We started those measurements at a pion energy of about 70 MeV because there exist differential cross section data to be compared with and because measurements are getting more and more difficult with decreasing pion energies due to the increasing energy loss of the pions in the necessarily relatively thick polarized target and because of the worse energy or momentum resolution, which hinders the separation of the scattering on the various nuclei of the complex target.

The experiment has been carried out at the \( \pi E 3 \) channel of the Paul Scherrer Institut with the low energy magnetic spectrometer LEPS, which features excellent momentum resolution (\( \approx 0.2\% \)) and effective muon suppression. The spectrometer has been described in details in Ref. [17]. Two beam defining scintillators have been mounted in front of the target to define the beam size corresponding to the size of the polarized target. The target material consisted of 95\% butanol and 5\% water. For the first data taking runs it was doped with 1\% porphyrin, later on with 1.5\% EHBA-Cr complexes. The target with a volume of \( 18 \times 18 \times 3 \text{ mm}^3 \) has been cooled down to 70 mK by a \( ^3\text{He}-^4\text{He} \) dilution refrigerator [19] and has been polarized dynamically in the magnetic field of a Helmholtz coil of 2.5 T. A maximum polarization of 85\% for spin up and 72\% for spin down has been reached. The polarization is reversed by irradiating microwaves at the opposite edge of the electron spin resonance absorption line. In almost any material the maximum polarization that can be achieved is different for negative and positive spins. The reason for this is not known; different effects may play a role, for example, the shape of the electron spin resonance line resulting from the specific paramagnetic dopant and its concentration or the polarization process itself. The degree of polarization was measured by standard techniques, the continuous-wave nuclear magnetic resonance (NMR) method [20], where the integral of the measured NMR signals is proportional to the target polarization. An absolute gauge of the signals is reached in the thermal equilibrium of the system. In this case the degree of polarization is only given by the Boltzmann factor and therefore known for a certain temperature. For the gauge we have used a temperature of 2.17 K. The degree of polarization for such a temperature is nearly three orders of magnitude smaller than the degree we have found in the case of dynamic polarization. Around 100 of the thermal NMR signals have been added for each target in order to lower the statistical error. Together with the systematic error due to the determination of the integral of the signals and the uncertainty of the temperature measurement, a precision of \( \pm 5\% \) in the polarization gauge has been reached. After polarizing the protons the magnetic field has been reduced to 0.8 T (frozen spin mode). The final deflection angle of the direct pion beam in the magnetic field was then 13.5° (Fig. 1). The relaxation time of the polarization was about 500 h.

Measurements have been carried out at laboratory scattering angles of 40°, 50°, 60°, and 70° with both polarizations. For background subtraction data have been taken with a 1-mm and a 3-mm carbon sheet in the target cavity and with an empty target cavity. It turned out that it was sufficient to use the measurements with the 1-mm carbon sheet. A small correction for the different energy loss in the target was applied (Fig. 2).

The scattered particles have been identified by their time of flight (TOF) versus the rf of the cyclotron and by their TOF through the spectrometer determined with the two beam defining scintillators in front of the target and the trigger scintillator in the focal plane of LEPS. Muons arising from \( \pi \) decays inside LEPS have been removed by a consistency
check of the particle trajectory in the spectrometer. The most efficient cut has been due to the target coordinates. They have been obtained by a traceback of the particle trajectory from the coordinates measured at the intermediate focus of the spectrometer to the position of the scattering target. The quality of this traceback has been checked by plotting the difference of the distributions for spin up and spin down.

Since the scattering of pions on $^{12}$C and $^{16}$O has no polarization dependence, this difference just gives the pure proton distribution. It clearly reveals the dimensions of the polarized target.

The number of pions scattered on protons was normalized to the number of pions scattered on heavier nuclei of the complex target, mainly on the spin zero nuclei $^{12}$C and $^{16}$O. By this normalization most of the systematic errors cancel. This normalized number of scattered pions can be interpreted as the differential cross section at a certain target polarization ($d\sigma/d\Omega$)$_{pol}$ in arbitrary units. It depends linearly on the degree of the target polarization $P$:

$$\frac{d\sigma}{d\Omega}_{pol} = \left(1 + PA_x\right) \times \left(1 + PA_y\right),$$

with

$$P = \frac{N_p^\uparrow - N_p^\downarrow}{N_p^\uparrow + N_p^\downarrow}, \quad -1 \leq P \leq 1,$$

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with

$$P = \frac{N_p^\uparrow - N_p^\downarrow}{N_p^\uparrow + N_p^\downarrow}, \quad -1 \leq P \leq 1,$$

The two symbols denote data taken with two different polarized targets as explained in the text. Also shown are the KP phase-shift prediction, a prediction derived from a single energy partial-wave fit to the differential cross section data of Brack et al. [22] and a partial-wave fit to the data of this paper. Only the S31 partial wave has been fitted, keeping the P31 and P33 waves fixed at the values of the KP phase-shift analysis. The phases are listed in Table II.
TABLE I. Results of the $\pi^+\bar{p}$ analyzing power measurements at $p_{\pi,\text{lab}}=154.10\pm0.50$ MeV/c. The two data sets correspond to measurements with two different target compositions (as mentioned in the text), the target polarization of which has been calibrated separately. The errors of the data are essentially given by the error of the target polarization, which amounts to $\pm5\%$ and is dominated by the error of the thermal NMR signals (see the text). Additional systematic and statistical errors of the NMR signals in the polarized state of the targets are included, but almost negligible compared to the $\pm5\%$ error mentioned.

<table>
<thead>
<tr>
<th>$\theta_{\text{lab}}$</th>
<th>$\theta_{\text{c.m.}}$</th>
<th>$A_y$</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>41.22</td>
<td>40.07</td>
<td>0.386</td>
<td>$\pm0.021$</td>
</tr>
<tr>
<td>51.03</td>
<td>60.24</td>
<td>0.377</td>
<td>$\pm0.019$</td>
</tr>
<tr>
<td>60.80</td>
<td>71.07</td>
<td>0.331</td>
<td>$\pm0.026$</td>
</tr>
<tr>
<td>41.22</td>
<td>40.07</td>
<td>0.408</td>
<td>$\pm0.013$</td>
</tr>
<tr>
<td>51.03</td>
<td>60.24</td>
<td>0.379</td>
<td>$\pm0.014$</td>
</tr>
<tr>
<td>60.80</td>
<td>71.07</td>
<td>0.366</td>
<td>$\pm0.018$</td>
</tr>
<tr>
<td>70.53</td>
<td>81.54</td>
<td>0.284</td>
<td>$\pm0.030$</td>
</tr>
</tbody>
</table>

$(d\sigma/d\Omega)_0$ being the unpolarized differential cross section and $N_p^\uparrow$ and $N_p^\downarrow$ the numbers of target protons with spin up and with spin down.

Data have been taken at various values of the target polarization $P$ including zero. To extract the analyzing power $A_y$ a straight line has been fitted to $(d\sigma/d\Omega)_{\text{pol}}$ as a function of $P$. The analyzing power $A_y$ is then given by the parameters $a,b$ of the fit:

$$
\frac{d\sigma}{d\Omega}_{\text{pol}} = aP + b,
$$

$$
A_y = a/b.
$$

By comparison of Eqs. (4) and (5) one reads $a = (d\sigma/d\Omega)_0 A_y$, $b = (d\sigma/d\Omega)_0$. For details see Ref. [21].

### III. RESULTS AND DISCUSSION

Data taking runs have been carried out with two different targets. One target was doped with porphyrexide, the other with EHBA-Cr$^3$ complexes. Both data sets have an indepen-

TABLE II. Nuclear phase shifts in degrees of the curves in Fig. 3 and $\chi^2$ of the data points.

<table>
<thead>
<tr>
<th>Input</th>
<th>$S31$</th>
<th>$P31$</th>
<th>$P33$</th>
<th>$\chi^2/N_{\text{DF}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KP</td>
<td>$-6.96$</td>
<td>$-1.23$</td>
<td>$10.13$</td>
<td>$116.54/7$</td>
</tr>
<tr>
<td>Brack et al. prediction</td>
<td>$-5.37$</td>
<td>$-1.77$</td>
<td>$9.84$</td>
<td>$34.21/7$</td>
</tr>
<tr>
<td>PW fit ($S31$)</td>
<td>$-6.07$</td>
<td>(KP)</td>
<td>(KP)</td>
<td>$6.31/6$</td>
</tr>
<tr>
<td>PW fit ($P$ waves)</td>
<td>(KP)</td>
<td>$-4.02$</td>
<td>$17.95$</td>
<td>$8.69/5$</td>
</tr>
</tbody>
</table>

TABLE III. Partial-wave fit results: nuclear phase shifts in degrees and scaling factors for the various data sets.

<table>
<thead>
<tr>
<th>$S31$</th>
<th>$P31$</th>
<th>$P33$</th>
<th>$\chi^2/N_{\text{DF}}$</th>
<th>Reference</th>
<th>Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-6.22\pm0.10$</td>
<td>$-1.25\pm0.13$</td>
<td>$9.59\pm0.07$</td>
<td>$13.95/13$</td>
<td>this work</td>
<td>$\ast1.05$</td>
</tr>
<tr>
<td>$-6.30\pm0.07$</td>
<td>$-1.19\pm0.08$</td>
<td>$9.68\pm0.10$</td>
<td>$61.84/58$</td>
<td>this work</td>
<td>$\ast1.05$</td>
</tr>
<tr>
<td>$-5.62\pm0.06$</td>
<td>$-1.74\pm0.06$</td>
<td>$9.92\pm0.04$</td>
<td>$22.32/26$</td>
<td>this work</td>
<td>$\ast0.95$</td>
</tr>
<tr>
<td>$-6.40\pm0.11$</td>
<td>$-0.46\pm0.12$</td>
<td>$10.2\pm0.1$</td>
<td>$44.06/14$</td>
<td>this work</td>
<td>$\ast0.95$</td>
</tr>
<tr>
<td>$-6.02\pm0.05$</td>
<td>$-1.50\pm0.05$</td>
<td>$9.70\pm0.03$</td>
<td>$157.79/80$</td>
<td>this work</td>
<td>$\ast1.05$</td>
</tr>
</tbody>
</table>

* More detailed information can be found in the references provided.
dent systematic error of ±5% that is due to the calibration of the target polarization. Figure 4 and Table I show the resulting analyzing powers as a function of the scattering angle. Since the data lie substantially beneath the KP phase-shift predictions, we have carried out an independent phase-shift analysis. If the $P_{31}$ and $P_{33}$ waves are fixed at the values of the KP phase-shift analysis [1] while the $S_{31}$ phase is varied freely, a fit to the data points results in $-6.07° ± 0.09°$ for the $S_{31}$ phase shift of the KP value and the $P$ waves are fitted freely to the data (see Table II). Hence we conclude that the $S_{31}$ phase shift of the KP analysis has to be modified in order to reproduce the data. In a more extensive analysis the $A_y$ data have been fitted together with the differential cross-section data of various authors [13,22–24]. Hereby least-squares fits have been carried out for all phases ($S_{31}, P_{31}, P_{33}$) with some overall rescaling of the various data sets. The data have been re-scaled either by up to ±5% or by the systematic errors of the data as quoted by the authors if they exceeded ±5%. The resulting phase shifts clearly depend somewhat on the data included in the fit, but generally speaking the absolute value of the $S_{31}$ phase shift is about 1° smaller than in the KP phase-shift analysis (Table III).

The efforts of recent years on the experimental side fortunately result in a more and more converging data base. The main result is that the $S_{31}$ and $S_{11}$ phases below $T_x = 100$ MeV are smaller than given by the KP analysis, which is supported by the first measurement of the analyzing power at those low energies described in the present paper. Some older data simply have to be disregarded in a new phase-shift analysis, which is extremely timely and has to use indispensably dispersion relations as theoretical constraints. This lack of a new dispersion analysis in the spirit of the KPH analysis including recently measured $\pi N$ data makes it difficult at present to draw final conclusions on a “new” value of the amplitude $\Sigma$.

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